Pumped spin and charge currents from applying a microwave field to a quantum dot between two magnetic leads

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The evolution-operator approach is applied to studying photon-electron-pumping effects on a quantum dot connected to two magnetic leads in the presence of both via-dot and over-dot tunneling channels. It is found that a microwave field applied to the quantum dot may give rise to charge and spin pumpings at zero-bias voltage for asymmetric magnetic junctions. More interestingly, a pure spin current can be pumped for symmetric magnetic junctions in the antiparallel magnetization configuration, providing an idea for the design of spin batteries.

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Charge and spin are two elementary degrees of freedom of an electron. While traditional electronics is based on the charge degree of freedom, the new and emerging technology of spintronics[1,2] strives to utilize not only the charge but also the spin degrees of freedom for electronic applications. For spin-polarized currents, if the spin-up and -down currents are along the same direction, the charge current is always greater than the spin current. If they have opposite directions, the charge current is smaller than the spin current. Recently, there has been much interest in the generation of a pure spin current without any net charge transport, for which many proposals have been made by means of spin batteries.[3–8] The spin battery is the analog of a charge battery in conventional electronics, but may have either one or two poles.[9] The dipole spin battery[7,8] is of particular interest because it can complete a spin-current circuit.

The coherent transport of electrons under the influence of time-dependent external fields has attracted increased interest due to the rapid development of fabricating nanoscale devices. The photon-assisted tunneling through a small quantum dot (QD) is an inelastic tunneling process with electrons exchanging energy with the oscillating field.[10] In the system where a QD is coupled to left or right leads, cyclic gate voltages applied to the QD and/or the microwave radiation on one lead can produce photon-electron-pumped current. While earlier studies of the QD-based pumping effect focused on the charge pumping,[11] these have been recently extended to the QD-based spin pumping.[12–15] Theoretically, Mucciolo *et al.*[12] proposed that a QD-based charge pump, consisting of an open QD driven by two ac (radio-frequency) gate voltages, can function as a phase-coherent spin pump in the presence of sizable Zeeman splitting. Experimentally, Watson *et al.*[13] demonstrated a mesoscopic spin pump using an ac-driven phase-coherent QD in a Zeeman field, including the ability to pump pure spin without any charge current. Most of the QD-based spin pumps were designed by use of nonmagnetic systems plus the Zeeman field, although a magnetic tunnel junction (no QD) by means of adiabatic quantum pumping was studied very recently.[15] The QD-based spin pumping has not been seriously investigated until now in the magnetic systems. At the same time, it is highly desirable to design a device with structure as simple as possible to construct a spin-pumped battery.

In this work we propose a scheme of the dipolar spin battery that consists of a common magnetic tunnel junction with a QD subject to microwave field. For asymmetric tunneling probabilities for the spin-s electronic channel between the QD and two leads, the microwave field applied to the QD can induce a difference in the chemical potential between the two leads. If a via-dot tunneling channel is present, as modeled on an external circuit, a spin-s current will tunnel from one lead to the other even at zero-bias voltage. In the antiparallel (AP) magnetization configuration, the spin-up and -down-pumped currents will flow in opposite directions, so that a pure spin current may be generated without any charge current.

Consider a magnetic tunnel junction with a QD subject to a microwave field and connected to two ferromagnetic metallic leads. There is an over-dot tunneling channel between the two leads as well as a via-dot channel between the QD and leads. The system can be described by Hamiltonian $H = H_0(t) + V$, with

$$H_0(t) = \sum_{\mathbf{k}_\alpha s} \varepsilon_{\mathbf{k}_\alpha s}(t) a^\dagger_{\mathbf{k}_\alpha s} a_{\mathbf{k}_\alpha s} + \varepsilon_d(t) a^\dagger_{ds} a_{ds}, \tag{1}$$

$$V = \sum_{\mathbf{k}_\alpha s} V_{\mathbf{k}_\alpha d} a^\dagger_{\mathbf{k}_\alpha s} a_{ds} + \sum_{\mathbf{k}_L \mathbf{k}_R s} V_{\mathbf{k}_L \mathbf{k}_R} a^\dagger_{\mathbf{k}_L s} a_{\mathbf{k}_R s} + \text{H.c.} \tag{2}$$

Here the operators $a^\dagger_{\mathbf{k}_\alpha s}$ ($a_{\mathbf{k}_\alpha s}$) and $a^\dagger_{ds}$ (a_{ds}) are the creation (annihilation) operators for the spin-s electrons in lead α ($\alpha = L, R$) and in the QD, respectively. $\varepsilon_{\mathbf{k}_\alpha s} = \varepsilon_{\mathbf{k}_\alpha} - sM_\alpha$, with M_α as the magnetization of lead α and $s = 1$ (-1) for the spin parallel (P) (antiparallel) to $\mathbf{M}_\alpha$. $\varepsilon_d(t) = \varepsilon_d + \Delta_d \cos \omega t$, where ε_d is the single energy level of the QD, and ω and Δ_d are the frequency and amplitude of the microwave field applied to the QD, respectively. The Coulomb interaction on the QD is not considered here, but it does not change the qualitative result of this work. $V_{\mathbf{k}_\alpha d}$ is the tunneling coefficient for electron tunneling between lead α and the QD, and $V_{\mathbf{k}_L \mathbf{k}_R}$ is that between the left and right leads.

The time-evolution operator $U(t, 0)$ is used to describe the dynamical evolution of charge and spin currents. In the interaction picture, $U(t, 0)$ satisfies the following equation:

$$i\hbar \frac{\partial}{\partial t} U(t,0) = \widetilde{V}(t) U(t,0), \tag{3}$$

where

$$\widetilde{V}(t) = U_0(t,0) V U_0^{\dagger}(t,0),$$

with

$$U_0(t,0) = T \exp[i \int_0^t dt_1 H_0(t_1)].$$

It has been assumed here that $V_{\vec{k}_\alpha d}=0$ and $V_{\vec{k}_L \vec{k}_R}=0$ before time $t=0$ and are constant interactions after time $t=0$. For simplicity, we set $\hbar=e=1$.

By use of the time-evolution-operator approach,[16–18] the tunneling current through the QD can be obtained in terms of the appropriate matrix elements of $U(t,0)$. The number of the spin-s electrons in the left lead is given by

$$\begin{aligned} n_{Ls}(t) &= \sum_{\mathbf{k}_L} n_{\mathbf{k}_L s}(t) \\ &= \sum_{\mathbf{k}_L} \Bigg[n_{ds}(0) |U^s_{\mathbf{k}_L d}(t,0)|^2 + \sum_{\mathbf{q}_L} n_{\mathbf{q}_L s}(0) |U^s_{\mathbf{k}_L \mathbf{q}_L}(t,0)|^2 \\ &\quad + \sum_{\mathbf{k}_R} n_{\mathbf{k}_R s}(0) |U^s_{\mathbf{k}_L \mathbf{k}_R}(t,0)|^2 \Bigg], \end{aligned} \tag{4}$$

where

$$U^s_{\mathbf{k}_L d}(t,0) = \langle \mathbf{k}_L s | U(t,0) | ds \rangle,$$

$$U^s_{\mathbf{k}_L \mathbf{q}_L}(t,0) = \langle \mathbf{k}_L s | U(t,0) | \mathbf{q}_L s \rangle,$$

and

$$U^s_{\mathbf{k}_L \mathbf{k}_R}(t,0) = \langle \mathbf{k}_L s | U(t,0) | \mathbf{k}_R s \rangle$$

stand for the matrix elements of $U(t,0)$ with $|\mathbf{k}_L s\rangle$, $|\mathbf{q}_L s\rangle$, ($|\mathbf{k}_R s\rangle$), and $|\mathbf{d}s\rangle$ as the electronic wave functions of the left (right) lead and QD, respectively. $n_{ds}(0)$ and $n_{\mathbf{k}_\alpha s}(0)$ are the initial occupation numbers of the corresponding single-particle states. The spin-s tunneling current from the left lead into the QD and the right lead can be obtained from the time derivative of $n_{Ls}(t)$, $j_s(t)=-dn_{Ls}(t)/dt$. In order to get $j_s(t)$, it is necessary to calculate the matrix elements of $U(t,0)$ that appear in Eq. (4). Following the time-evolution-operator approach,[16–18] we can get coupled integro-differential equations for these matrix elements. Under the wide-band-limit (WBL) approximation,[16–18] after a lengthy calculation, the required matrix elements of the evolution operator are obtained as

$$U^s_{\mathbf{k}_L d}(t,0) = -F_{Rs} \int_0^t dt_1 \widetilde{V}^s_{\mathbf{k}_L d}(t_1) \exp(-C_s t_1),$$

$$\begin{aligned} U^s_{\mathbf{k}_L \mathbf{q}_L}(t,0) &= \delta_{\mathbf{k}_L \mathbf{q}_L} \\ &\quad - \frac{\pi V_{LR}^2 / B_{Rs}}{1+\chi^2} \frac{\exp[i(\varepsilon_{\mathbf{k}_L s} - \varepsilon_{\mathbf{q}_L s})t] - 1}{i(\varepsilon_{\mathbf{k}_L s} - \varepsilon_{\mathbf{q}_L s})} \\ &\quad - F_{Rs} \int_0^t dt_1 \widetilde{V}^s_{\mathbf{k}_L d}(t_1) U^s_{d\mathbf{q}_L}(t_1,0), \end{aligned}$$

$$\begin{aligned} U^s_{\mathbf{k}_L \mathbf{k}_R}(t,0) &= -F_{Rs} \int_0^t dt_1 \widetilde{V}^s_{\mathbf{k}_L d}(t_1) U^s_{d\mathbf{k}_R}(t_1,0) \\ &\quad - \frac{V_{LR}}{1+\chi^2} \frac{\exp[i(\varepsilon_{\mathbf{k}_L s} - \varepsilon_{\mathbf{k}_R s})t] - 1}{\varepsilon_{\mathbf{k}_L s} - \varepsilon_{\mathbf{k}_R s}}, \end{aligned}$$

with

$$U^s_{d\mathbf{k}_\alpha}(t,0) = -F_{\alpha s} \int_0^t dt_1 \widetilde{V}^{s*}_{\mathbf{k}_\alpha d}(t_1) \exp[-C_s(t-t_1)],$$

$$\widetilde{V}^s_{\mathbf{k}_\alpha d}(t) = V_{\alpha d} \exp[it(\varepsilon_{\mathbf{k}_\alpha s} - \varepsilon_d) - i\Delta_d \sin \omega t/\omega].$$

Here

$$F_{\alpha s} = \left(i + \tfrac{1}{2}\Gamma^s_\alpha V_{LR}/V^2_{\alpha d}\right)/(1+\chi^2)$$

and

$$C_s = \left(\tfrac{\Gamma^s_L + \Gamma^s_R}{2} - i\chi\sqrt{\Gamma^s_L \Gamma^s_R}\right)/(1+\chi^2),$$

with

$$\chi = \sqrt{\Gamma^s_L \Gamma^s_R} V_{LR}/(2V_{Ld}V_{Rd}),$$

$\Gamma^s_\alpha = 2\pi V^2_{\alpha d}/B_{\alpha s}$ denoting the spin-s linewidth function of lead α, and $B_{\alpha s}$ as the effective bandwidth of spin-s electrons in lead α or the inverse density of states $1/\rho_{\alpha s}$. It has been assumed that $V_{\mathbf{k}_\alpha d} \equiv V_{\alpha d}$ and $V_{\mathbf{k}_L \mathbf{k}_R} \equiv V_{LR}$ with $\alpha=L$ and R, independent of $\mathbf{k}$.[16–18] The hybridization matrix elements V_{LR} responsible for the direct tunneling channel, as well as Γ^s_α for the tunneling channel between the QD and lead α, play an important role in the pumped charge and spin currents.

Substituting these matrix elements of $U(t,t_0)$ into Eq. (4) and using $j_s(t)=-dn_{Ls}(t)/dt$, we can get the tunneling current $j_{Ls}(t)$ in an analytical form.[16–18] Under the assumption of $V_{Ld}=V_{Rd}$,[19] the spin-s tunneling current $j_s(t)$ at zero-bias voltage and at zero temperature is obtained as

$$\begin{aligned} j_s(t) &= \frac{\pi \Gamma_s \rho_{Ls} \rho_{Rs} V_{LR}^2}{(1+\chi^2)^4} (\Gamma^s_L - \Gamma^s_R) \\ &\quad \times \int_{-\infty}^{\mu} [|D_{Ls}(\varepsilon,t)|^2 - |E_{Ls}(\varepsilon,t)|^2] d\varepsilon, \end{aligned} \tag{5}$$

where

$$\begin{aligned} D_{Ls}(\varepsilon,t) &= -i \int_0^t \exp[i(t-t_1)(\varepsilon - \varepsilon_d) \\ &\quad - i\Delta_d(\sin \omega t - \sin \omega t_1)/\omega - C_s(t-t_1)] dt_1, \end{aligned}$$

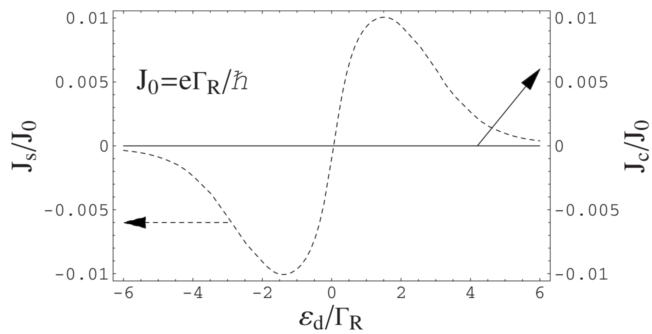

FIG. 1. Spin current J_s (dashed line) and charge current J_c (solid line) as functions of the QD energy level ε_d in the AP magnetization configuration with $\xi=0.5$, $V_{LR}=10$, $V_{Ld}=V_{Rd}=4$, $\Delta_d=4$, and $\omega=1$ with $\Gamma_L=\Gamma_R=1$ as the unit of energy.

$$E_{Ls}(\varepsilon,t) = -i\int_0^t \exp[i(t-t_1)(\varepsilon-\varepsilon_d) - C_s(t-t_1)]dt_1,$$

with $\Gamma_s=\Gamma_L^s+\Gamma_R^s$. Here the asymmetry of Γ_L^s and Γ_R^s arises from the difference in the spin-dependent density of states $\rho_{\alpha s}$ between the left and right leads, resulting in nonzero pumped $j_s(t)$. In the case of $\Gamma_L^s=\Gamma_R^s$, $j_s(t)$ is always vanishing. On the other hand, since $j_s(t)\propto V_{LR}^2$, a nonzero over-dot tunneling probability is necessary for a nonzero pumped current to be obtained.

In the present work we focus our attention on the time average of $j_s(t)$ at zero temperature. Since $j_s(t)$ is a temporal periodic function with period $2\pi/\omega$, its average can be defined as

$$\langle j_s(t)\rangle = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega} j_{Ls}(t)dt. \tag{6}$$

In the expression for $D_s(\varepsilon,t)$, $\exp(-\Delta_d \sin \omega t_1/\omega)$ can be expanded in series in terms of Bessel functions. After performing all the integrals over times and energy, we obtain the time average spin-s tunneling current as

$$\langle j_s(t)\rangle = \frac{2\chi^2(\Gamma_L^s-\Gamma_R^s)}{\pi(1+\chi^2)^3}\sum_{m=-\infty}^{+\infty}\left[\delta_{m0} - J_m^2\left(\frac{\Delta_d}{\omega}\right)\right] \times\arctan[2(\omega m+\eta)(1+\chi^2)/\Gamma_s], \tag{7}$$

with

$$\eta = \varepsilon_d - \mu - \chi\sqrt{\Gamma_L^s\Gamma_R^s}/(1+\chi^2)$$

and $J_m(\Delta_d/\omega)$ as the mth-order Bessel function. The derivation of Eq. (7) from Eq. (5) is given in the Appendix. Equation (7) is one of the main results of this work. At zero-bias voltage, we obtain the pumped charge current as $J_c=\langle j_\uparrow(t)\rangle+\langle j_\downarrow(t)\rangle$ and the pumped spin current as $J_s=\langle j_\uparrow(t)\rangle-\langle j_\downarrow(t)\rangle$. It follows from Eq. (7) that at least three factors are necessary to give rise to pumped currents. The first factor is the presence of the over-dot tunneling channel (nonzero V_{LR}), for there is a prefactor χ^2 proportional to V_{LR}^2 in Eq. (7). The second one is the presence of the microwave field applied to the QD. If either $\Delta_d=0$ or $\omega=0$, there would be a vanishing $\langle j_s(t)\rangle$ in Eq. (7). The third one is the asymmetry of Γ_L^s and Γ_R^s, for $\langle j_s(t)\rangle$ is proportional to the difference between them.

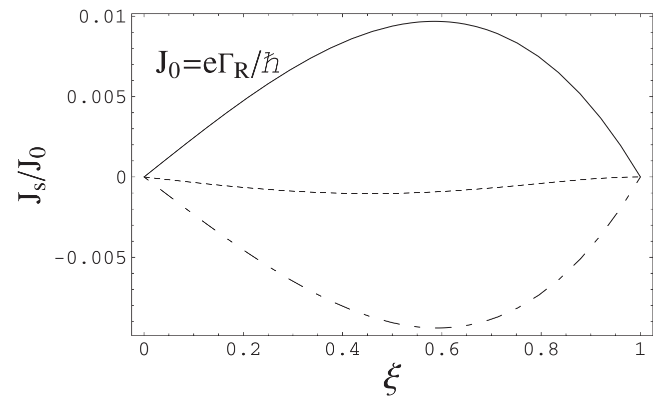

FIG. 2. Spin current J_s as a function of polarization ξ for QD energy level $\varepsilon_d=2$ (solid line), 0 (dashed line), and -2 (dot-dashed line) in the AP magnetization configuration. The other parameters are the same as those in Fig. 1.

If both magnetic leads are made of the same ferromagnetic metal ($\rho_{Ls}=\rho_{Rs}$) and they are symmetric about the QD ($V_{Ld}=V_{Rd}$), we have $\Gamma_L=\Gamma_R$ with $\Gamma_\alpha=\Gamma_\alpha^\uparrow+\Gamma_\alpha^\downarrow$. In the P magnetization configuration, $\Gamma_L^s=\Gamma_R^s$ so that there is neither charge pumping nor spin current pumping. A most interesting result can be obtained in the AP case, in which $\Gamma_L^\uparrow=\Gamma_R^\downarrow$ and $\Gamma_L^\downarrow=\Gamma_R^\uparrow$. In this case, we obtain $\langle j_\uparrow(t)\rangle=-\langle j_\downarrow(t)\rangle$ proportional to $\Gamma_L^\uparrow-\Gamma_L^\downarrow$, so that there is a pumped spin current J_s,

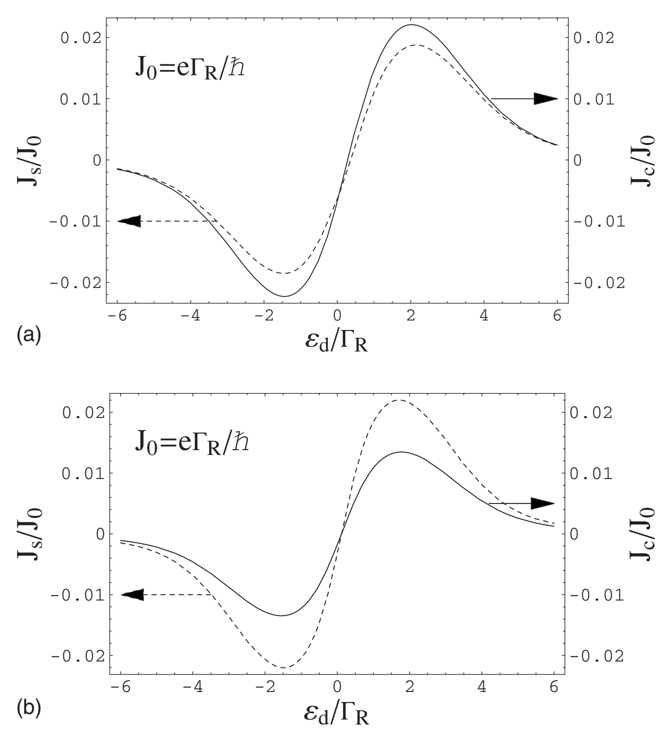

FIG. 3. Spin current J_s as a function of the QD energy level ε_d in the (a) P and (b) AP configurations with $\Gamma_L=2$ and $\Gamma_R=1$. The other parameters are the same as those in Fig. 1.

even though the net charge current $J_c=0$. In the present device, such a pumping of the microwave field can result in a pure spin current, providing an idea for the design of spin batteries. In what follows we present some numerical results to show the physical picture of J_s. We define the spin polarization of two ferromagnetic leads as $\xi=\xi_\sigma=|\rho_{\alpha\uparrow}-\rho_{\alpha\downarrow}|/\rho_\alpha$, with $\rho_\alpha=\rho_{\alpha\uparrow}+\rho_{\alpha\downarrow}$, so that we have $\rho_{\alpha s}=\rho_\alpha(1+s\xi)$ in the P case, and $\rho_{Ls}=\rho_L(1+s\xi)$ and $\rho_{Rs}=\rho_R(1-s\xi)$ in the AP case. Figure 1 shows pumped spin current J_s/J_0 varying with energy level ε_d of the QD, where $J_0=e\Gamma_R/\hbar$. It is found that J_s is an antisymmetric function of ε_d since J_s remains unchanged in magnitude but changes its sign when η in Eq. (7) is replaced with $-\eta$. It then follows from $\eta=0$ that the antisymmetric center of J_s is at $\varepsilon_d=\mu+\chi\sqrt{\Gamma_L^s\Gamma_R^s}/(1+\chi^2)$. On the other hand, the variation in J_s with ξ is nonmonotonous, as shown in Fig. 2. This behavior is the result of competition between two factors. The prefactor of Eq. (7) contains a product of $\Gamma_\uparrow-\Gamma_\downarrow$ and χ^2, the former increasing linearly with ξ and the latter being proportional to $1-\xi^2$. As a result, the maximal J_s should be around $\xi=\sqrt{1/3}$, consistent with that shown in Fig. 2.

If the left and right leads are made of different ferromagnetic metals ($\rho_{Ls}\neq\rho_{Rs}$), we have $\Gamma_L^s\neq\Gamma_R^s$ and $\Gamma_L^s\neq\Gamma_R^{-s}$, so that there is a pumped charge current, in either P or AP configuration, which is spin polarized. We also present some numerical results of J_s and J_c. The parameters used are the same as in Fig. 1 except that $\Gamma_L=2$ and $\Gamma_R=1$ are taken. Figures 3(a) and 3(b) show J_s (J_c) as a function of ε_d in the P and AP configurations, respectively. In the P case, since $\langle j_\uparrow(t)\rangle$ and $\langle j_\downarrow(t)\rangle$ are along the same direction, J_c is greater than J_s in magnitude. In the AP case, the situation is just the opposite since $\langle j_\uparrow(t)\rangle$ and $\langle j_\downarrow(t)\rangle$ have different directions. For nonmagnetic leads, provided that $\rho_L\neq\rho_R$, the pumping of the microwave field applied to the QD can result in a charge current, forming a charge battery.

Finally, we wish to discuss the physical mechanism of the photopumped charge and spin currents in the absence of external voltage applied between two leads. The microwave field applied to the QD is the power supply source, which can result in a small difference in the chemical potential between the two leads of the magnetic tunnel junction. This potential difference arises from the asymmetry of $\Gamma_L^\uparrow\neq\Gamma_R^\downarrow$, no matter whether the over-dot tunneling exists or not. For ε_d below μ, the electron occupying the ε_d level of the QD may tunnel into two leads under the action of the microwave field. Owing to asymmetric Γ_L^s and Γ_R^s, the probabilities for the spin-s electron to tunnel into the left and right leads are different, leading to a small difference in the chemical potential between the two leads. If ε_d is above μ, there is no electron on the ε_d level. This situation can be regarded as a hole occupying the ε_d level of the QD. From a similar analysis, it follows that there exists a small difference in the chemical potential between the left and right leads, which has a sign opposite to that in the electron case. This can explain why the pumped current exhibits antisymmetric behavior with respect to $\varepsilon_d=\mu+\chi\sqrt{\Gamma_L^s\Gamma_R^s}/(1+\chi^2)$ in Figs. 1 and 3. For $V_{LR}=0$, the present junction is an open spin battery, in which the two leads have spin-dependent potential difference, but the pumped current is absent ($J_s=J_c=0$). The role of the over-dot tunneling is to provide a passageway of spin currents for the spin battery, like an external circuit of a common battery. In the presence of an over-dot tunnel channel, a pumped spin and/or charge current would be formed from the higher chemical potential to the lower one. As a result, the over-dot tunneling is necessary to obtain the tunneling (spin or charge) current, rather than the spin battery effect.

The idea of forming a spin battery is especially interesting. Such a spin battery is simply constituted by a magnetic tunnel junction with a QD subject to microwave fields, in which spin polarization ξ of the magnetic leads does not need to be very high. Since a symmetric structure has been assumed (i.e., $\Gamma_L^\uparrow=\Gamma_R^\uparrow$ and $\Gamma_L^\downarrow=\Gamma_R^\downarrow$), the antiparallel magnetization configuration of the two leads is necessary for obtaining the spin battery effect. At the same time, the two leads must be ferromagnetic. If spin polarization $\xi=0$ (nonmagnetic), there is no spin battery effect. On the other hand, for $\xi=1$ (half metallic), there is a maximal spin-dependent potential difference, but there is no spin current in the two spin-channel model without spin flip. In both cases, $J_s=0$ as shown in Fig. 2. For $0<\xi<1$, the competition between the two factors results in a nonmonotonous change in J_s with ξ in Fig. 2. The proposed spin battery for producing the spin current should be experimentally feasible using the present technology. First, the QD structures can be fabricated in laboratories. Second, the microwave-pumped quantum transport measurements have already been reported. In particular, the microwave radiation on the QD device has already been carried out experimentally.[20] In the present calculation $\Gamma_R=1$ meV and $\omega=1$ are taken so that $f=\omega/2\pi=250$ GHz, which is in the microwave range. The maximal spin current in Fig. 1 is equal to about $0.01e\Gamma_R/\hbar\simeq2.5$ nA.

In summary, we have studied charge and spin transport through a QD subject to a microwave field and coupled to two magnetic leads in the presence of the over-dot tunneling. The time-dependent tunneling current and the average tunneling current are obtained by the evolution-operator approach. It is found that at zero-bias voltage and in the AP configuration of the magnetic junction, the microwave field applied to the QD can give rise to a pure spin current. This result can be used in designing an alternative type of spin battery.

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APPENDIX: DERIVATION OF EQ. (7) FROM EQ. (5)

Starting from Eq. (5), together with the expressions for $D_{Ls}(\varepsilon,t)$ and $E_{Ls}(\varepsilon,t)$, and using the relation $\exp[ix\sin\varphi]=\Sigma_{m=-\infty}^{+\infty}J_n(x)\exp[in\varphi]$, we have

$$
\begin{aligned}
|D_{Ls}(\varepsilon,t)|^2 &= D_{Ls}(\varepsilon,t)D_{Ls}^*(\varepsilon,t)\\
&= \exp\left[\frac{-\Gamma_s t}{1+\chi^2}\right]\sum_{n=-\infty}^{+\infty} J_n\left(\frac{\Delta_d}{\omega}\right)\int_0^t \exp\left[\left\{i\Lambda_n + \frac{\Gamma_s/2}{1+\chi^2}\right\}t_1\right]dt_1\\
&\quad\times \sum_{m=-\infty}^{+\infty} J_m\left(\frac{\Delta_d}{\omega}\right)\int_0^t \exp\left[\left\{-i\Lambda_m + \frac{\Gamma_s/2}{1+\chi^2}\right\}t_2\right]dt_2\\
&= \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} J_n\left(\frac{\Delta_d}{\omega}\right)J_m\left(\frac{\Delta_d}{\omega}\right)\frac{\exp[i(n-m)\omega t]}{\left[-i\Lambda_m+\frac{\Gamma_s/2}{1+\chi^2}\right]\left[i\Lambda_n+\frac{\Gamma_s/2}{1+\chi^2}\right]},
\end{aligned}
$$

where

$$
\Lambda_n = n\omega + \varepsilon_d - \varepsilon - \frac{\chi\sqrt{\Gamma_L^s\Gamma_R^s}}{1+\chi^2}.
$$

In the last step of the above calculation, since some terms tend to zero for $t\to\infty$, they have been omitted. From Eq. (6), the time average of $|D_{Ls}(\varepsilon,t)|^2$ can be obtained as

$$
\begin{aligned}
\langle|D_{Ls}(\varepsilon,t)|^2\rangle &= \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} J_n\left(\frac{\Delta_d}{\omega}\right)J_m\left(\frac{\Delta_d}{\omega}\right)\int_{-\pi/\omega}^{\pi/\omega}\frac{\omega\exp[i(n-m)\omega t]}{2\pi\left[-i\Lambda_m+\frac{\Gamma_s/2}{1+\chi^2}\right]\left[i\Lambda_n+\frac{\Gamma_s/2}{1+\chi^2}\right]}dt\\
&= \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} J_n\left(\frac{\Delta_d}{\omega}\right)J_m\left(\frac{\Delta_d}{\omega}\right)\frac{\delta_{mn}}{\left[-i\Lambda_m+\frac{\Gamma_s/2}{1+\chi^2}\right]\left[i\Lambda_n+\frac{\Gamma_s/2}{1+\chi^2}\right]}\\
&= \sum_{n=-\infty}^{+\infty} J_n^2\left(\frac{\Delta_d}{\omega}\right)\frac{1}{\Lambda_n^2+\left(\frac{\Gamma_s/2}{1+\chi^2}\right)^2}.
\end{aligned}
$$

By the similar approach, the time average of $|E_{Ls}(\varepsilon,t)|^2$ can be obtained as

$$
\langle|E_{Ls}(\varepsilon,t)|^2\rangle = \frac{1}{\Lambda_0^2+\left(\frac{\Gamma_s/2}{1+\chi^2}\right)^2}.
$$

Substituting the results for $\langle|D_{Ls}(\varepsilon,t)|^2\rangle$ and $\langle|E_{Ls}(\varepsilon,t)|^2\rangle$ into Eq. (6), we obtain the time average of $j_s(t)$ as

$$
\begin{aligned}
\langle j_s(t)\rangle &= \frac{\pi\Gamma_s\rho_{Ls}\rho_{Rs}V_{LR}^2}{(1+\chi^2)^4}(\Gamma_L^s-\Gamma_R^s)\int_{-\infty}^{\mu}(\langle|D_{Ls}(\varepsilon,t)|^2\rangle-\langle|E_{Ls}(\varepsilon,t)|^2\rangle)d\varepsilon = \frac{\pi\Gamma_s\rho_{Ls}\rho_{Rs}V_{LR}^2}{(1+\chi^2)^4}(\Gamma_L^s-\Gamma_R^s)\sum_{m=-\infty}^{+\infty}\left[\delta_{m0}-J_m^2\left(\frac{\Delta_d}{\omega}\right)\right]\\
&\quad\times\left\{\arctan[2(\omega m+\eta)(1+\chi^2)/\Gamma_s]-\frac{\pi}{2}\right\} = \frac{2\chi^2(\Gamma_L^s-\Gamma_R^s)}{\pi(1+\chi^2)^3}\sum_{m=-\infty}^{+\infty}\left[\delta_{m0}-J_m^2\left(\frac{\Delta_d}{\omega}\right)\right]\arctan[2(\omega m+\eta)(1+\chi^2)/\Gamma_s],
\end{aligned}
$$

which is just the result of Eq. (7). Here the relation $\Sigma_{m=-\infty}^{+\infty}[\delta_{m0}-J_m^2(\frac{\Delta_d}{\omega})]=0$ has been used.

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